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Fracture Mechanics as Related to the Neuber Stress Concentration Factor and Plastic Particle

J. A. KIES AND H. L. SMITH

Ocean Materials Criteria Branch
Ocean Technology Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

ABSTRACT

A previously proposed relation between the Irwin stress intensity factor K and the Neuber stress concentration factor was examined to show the degree of its realism and applicability. Realistic limits to the stress σ_{max} and the radius of curvature of the notch ρ were stated for that relation. The radius of curvature R at the tip of a totally elastic crack having strain energy equivalent to corresponding realistic limit conditions of the proposed relation was derived in terms of K. The Neuber plastic particle size was shown to be four times the Irwin plastic zone correction regardless of the conditions of notch constraint. The elastic crack opening was discussed from the viewpoint of an experimentalist. A separate formula for the plastic crack opening displacement was derived based on plastic work. The total displacement, if measured, would be the sum of the elastic and plastic displacements, which are nearly equal. The formula derived for the plastic displacement was quite similar to the Wells $C.O.D.$, which was originally published in 1961 but derived from a different set of premises, namely, the purely elastic displacement at the particular position a_0 for the crack of depth $a_0 + r_y$.

PROBLEM STATUS

This is a final report on the topic indicated by the title. The work is part of a continuing program of fracture strength studies.

AUTHORIZATION

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FRACTURE MECHANICS AS RELATED TO THE NEUBER STRESS CONCENTRATION FACTOR AND PLASTIC PARTICLE

BACKGROUND

Neuber as quoted by Peterson (1) expressed the theoretical stress concentration factor k_t for a shallow-edge notch in tension as $k_t = 1 + 2\sqrt{a/\rho}$, where a is the notch depth and ρ is the root radius. For large values of a/ρ, i.e., high k_t we can write

$$k_t \approx 2\sqrt{\frac{a}{\rho}} = \frac{\sigma_y}{\sigma_0}\,, \tag{1}$$

where σ_y maximum is the stress on the surface at the root of the notch and σ_0 is the nominal applied stress.

In fracture mechanics the stress intensity factor K for a shallow-edge crack is given by

$$\frac{K}{\sqrt{\pi}} = 1.1\ \sigma_0\sqrt{a}\ . \tag{2}$$

By combining Eqs. 1 and 2 we have

$$\frac{K}{1.1\ \sqrt{\pi}} = \lim_{\rho\to 0}\left(\sqrt{\rho}\,\frac{\sigma_y^{\max}}{2}\right), \tag{3}$$

which is a slight modification of a corresponding formula given as Eq. 10 by Irwin (2). Equation 3 may be called an equivalence formula. For a certain useful but restricted range this was demonstrated by Sanford to be correct (3).

APPLICATIONS OF THE EQUIVALENCE FORMULA

The degree of utility of Eq. 3 was not discussed in Ref. 2. Since the upper limit must be imposed that $K = K_c$, we would have to assume that σ_y maximum can increase without limit as ρ approaches zero. Although it is easy to visualize ρ approaching zero, it is not realistic to say that σ_y maximum actually increases without limit in a metal. The term σ_y maximum can behave that way only if we relate it to a fictional crack, which has the same total elastic energy in the field as a real crack in which plasticity occurs at the crack tip.

The Irwin plasticity correction for plane stress is

$$\Delta a = r_y = \frac{K^2}{2\pi\sigma_{ys}^2} \tag{4a}$$

and for plane strain

$$\Delta a = r_y = \frac{K^2}{6\pi\sigma_{ys}^2}, \tag{4b}$$

where σ_{ys} is the uniaxial tensile yield strength. The plane strain formula is equivalent to the plane stress formula, if we assume a constraint factor f such that the value of σ for yielding is $f\sigma_{ys}$.

In this case we can let f take whatever value it must to represent mixed modes as well as pure plane stress and pure plane strain. For the Irwin plane strain case, $f^2 \approx 3$.

The plasticity correction Δa for the general case is then given by

$$\Delta a = \frac{K^2}{2\pi f^2 \sigma_{ys}^2} \,. \tag{4c}$$

Another point of interest in Eq. 3 is the question of what value to place on ρ for the hypothetical or fictional elastic equivalent to the real crack. The root radius ρ for an elastic crack depends on the applied stress and crack depth in a way which can be computed independent of the Neuber analysis.

The radius of curvature at the crack tip is that for an ellipse having a as the semiaxis (crack depth) and b as the semiopening (at the mouth). Therefore, the radius of curvature for the elastic case is given by

$$R = \frac{b^2}{a} \,. \tag{5}$$

The term b was given as η_0 in a paper by Irwin (4), from which

$$\eta_0^2 = 4(1-\nu^2)^2 \frac{\sigma_0^2 a^2}{E^2} = b^2 \,. \tag{6}$$

Therefore,

$$R = 4(1-\nu^2)^2 \frac{\sigma_0^2 a}{E^2} \,.$$

Since $K^2 = 1.21\pi\sigma_0^2 a$ for a surface crack (5),

$$R = \frac{4(1-\nu^2)^2 K^2}{E^2 \, 1.21\pi} \,.$$

For $\nu = 0.3$

$$R = \frac{K^2}{E^2}(0.87) \,. \tag{7}$$

If Eq. 3 is to represent a fictitious equivalent crack in which $\rho = R$, then

$$\frac{K}{1.1\sqrt{\pi}} = \lim_{\rho\to 0} \frac{\sigma_y^{max}}{2} \frac{2K(1-\nu^2)}{E \, 1.1\sqrt{\pi}}$$

and for that case

$$\lim_{\rho \to 0} \sigma_y^{max} = \frac{E}{(1-\nu^2)} \cdot \tag{8}$$

This is, of course, about ten times the "theoretical strength" of a solid. Therefore, the lower limit of ρ or R is set at $E/10$ or σ_{ys}, whichever is less.

Equation 8 is independent of K or σ_0 and is a concept, not a quantity to be measured. The elastic displacement of the "crack" boundary at the tip is always zero so that if elastic displacements are to be measured the easiest and most meaningful place would be at the specimen surface, where the total elastic opening for a shallow edge crack is (as given by Ref. 4)

$$2\eta_0 = 2b = 4(1-\nu^2)\frac{\sigma_0 a}{E} , \tag{9a}$$

or

$$K^2 = \frac{1.21\,\pi\sigma_0 E}{2(1-\nu^2)}\,\eta_0 \,. \tag{9b}$$

If we use the Irwin plasticity correction of Eq. 4c

$$2\eta_0 = \frac{4(1-\nu^2)}{E}\,\sigma_0\left(a_0 + \frac{K^2}{2\pi f^2 \sigma_0^2}\right). \tag{10}$$

Thus, K is determined by the elastic displacement at the surface as in Eqs. 9b and 10 but <u>not</u> by an elastic displacement at the tip.

THE NEUBER PLASTIC PARTICLE AND THE C.O.D.

A fictitious but conceptually interesting value of the average stress $\bar{\sigma}$ on the Neuber plastic particle is computed by assuming perfect linear elasticity as per Irwin (2), or

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} ,$$

where r is the distance beyond the end of a crack at a point on the x axis. Then

$$\bar{\sigma}_y = \frac{1}{\epsilon}\int^{\epsilon} \sigma_y \; dr \tag{11}$$

or

$$\bar{\sigma}_y = \frac{K}{\sqrt{\pi}}\sqrt{\frac{2}{\epsilon}} , \tag{12}$$

where ϵ is the Neuber plastic particle size (2).

If we now insert realism in the form of $\bar{\sigma}_y = f\sigma_{ys}$, using the definitions of f and σ_{ys} previously mentioned,

$$\epsilon = 4 \left(\frac{K^2}{2\pi f^2 \sigma_{ys}^2} \right) . \tag{13}$$

Thus, according to the above mathematical model, ϵ is four times the size of the Irwin plastic zone correction regardless of f. A direct correspondence is thus established but depends on certain somewhat artificial assumptions as noted above.

If the plastic displacement in the y direction is computed for crack propagation conditions in which we assume that the plastic strain in the plastic zone ϵ is p, the work done in creating the plastic deformation per unit volume of flowed metal is $p\bar{\sigma} = dW/dV$.

The element of volume is $dV = B\epsilon\ dx$, where ϵ is assumed to be the height as well as the extent of the Neuber plastic particle and B is the thickness of the section. Let

$$\mathcal{G}_c = \frac{dW}{Bdx} = p\bar{\sigma}\epsilon . \tag{14}$$

if

$$\frac{K_c^2}{E} = \mathcal{G}_c = \frac{dW}{dA} ,$$

then

$$\epsilon = \frac{K_c^2 \pi}{2p^2 E^2} . \tag{15}$$

Assuming that the plastic $C.O.D.$ at the crack tip is given by the plastic extension in the Neuber plastic particle and that ϵ is the dimension in the y direction as well as in the x direction, we get by definition, where p is the plastic strain,

$$C.O.D. = \delta_p = \epsilon p . \tag{16}$$

Rewriting Eq. 15 gives

$$p^2 = \frac{\pi K_c^2}{2E^2 \epsilon} \tag{17}$$

or

$$\delta_p = \frac{\pi K_c^2}{2E^2 p} \tag{18}$$

and

$$\epsilon = \frac{2K^2}{\pi f^2 \sigma_{ys}^2}$$

by combining Eqs. 13 and 17.

Rewriting Eq. 17 gives

$$p = \frac{\pi f \sigma_{ys}}{2E} . \tag{19}$$

Then

$$\delta_p = \frac{K_c^2}{E f \sigma_{ys}} \tag{20a}$$

or

$$\delta_p = \frac{\mathcal{G}}{f \sigma_{ys}} \; . \tag{20b}$$

INTERPRETATIONS OF FORMULAS FOR C.O.D.

Equation 20b gives the plastic $C.O.D.$ in terms of the uniaxial yield strength $\mathcal{G}$ and the constraint factor f. The result agrees with the usual formulas for the $C.O.D.$ (6,7). This agreement points up the equivalence of the usual derivations and the assumptions as made here with respect to the Neuber plastic particle. In the foregoing model the $C.O.D.$ would be measurable anywhere across gage points bridging the crack and plastic layer on either side. The $C.O.D.$ should be measured while the load is on and just at the onset of crack propagation if K_c is to be calculated from the $C.O.D.$ The total measured displacement will in that case include the elastic displacement as given by the Irwin formula (3). Thus, the total $C.O.D.$ is given by $\delta_p + 2\eta_0$ at the surface, and within the crack,

$$2\eta = 2\eta_0 \sqrt{\frac{1 - x^2}{a^2}} \; , \quad x \le a \; , \tag{21}$$

is the elastic component of the $C.O.D.$

Wells (6) was the first to compute a $C.O.D.$ using a mixture of elastic and plastic formulas. He wrote for the elastic displacement from the midplane at a distance r from the tip measured in the direction toward the mouth of the crack

$$\eta = \frac{2K}{E}\sqrt{\frac{2r}{\pi}} \; . \tag{22}$$

This is a fair approximation to the more exact expression in Eq. 21. Wells then proceeded to select a distance from the tip of the crack $r = r_y = \Delta a$ as given by the Irwin plasticity correction. At this point the elastic displacement 2η given by Eq. 22 is

$$2\eta = \frac{4K}{E}\sqrt{\frac{2\Delta a}{\pi}} \; .$$

If

$$\Delta a = \frac{K^2}{2\pi\sigma_{ys}^2} \; ,$$

then

$$2\eta = \frac{4K^2}{E\pi\sigma_{ys}} = \frac{4\mathcal{G}}{\pi\sigma_{ys}} \; . \tag{23}$$

Although the similarity between Wells' result as shown in Eq. 23 and that in Eq. 20 is striking, the rationale for Wells' choice of $r = r_y$ as the point at which a calculation of an elastic displacement is made and which is said to result from an Irwin plastic zone is not clear. It might be interpreted as the elastic displacement of a fictitious crack length $a + \Delta a$ at the point a_0 and in which

$$\Delta a = r_y = \frac{K^2}{2\pi\sigma_{ys}^2} .$$

The rationale used in arriving at Eq. 20 seems more clear but still subject to some doubt as to the realism or validity of the result. The result of Burdekin (7) giving $\delta = \mathcal{G}/\sigma_{ys}$ as the first term of a series expansion leaves something to be desired, since the series does not converge except under very severe restrictions.

LATERAL CONTRACTION AT THE CRACK TIP

Let the lateral plastic contraction at the tip of the crack be expressed as a fraction $p = -\Delta B/B_0$, where $-\Delta B$ is the contraction and B_0 is the initial plate thickness. For a constant volume of strained metal

$$p = \frac{-\Delta B}{B_0} = \frac{\Delta L}{L_0} .$$

It was shown in Eq. 19 that

$$p = \frac{\pi f \sigma_{ys}}{2E} ,$$

but this does not give K_c in terms of p independent of δ_p.

Assume $\mathcal{G}_c$ is due to plastic work; i.e.,

$$\mathcal{G}_c = f\sigma_{ys}(\epsilon p) ,$$

where ϵ is the height of the plastic zone and p is the strain.

We have shown that $\epsilon = 2K_c^2/\pi f^2\sigma_{ys}^2$. Therefore, measuring the height of the plastic zone would give K_c if we knew f and σ_{ys}.

The lateral contraction cannot be used to give $\mathcal{G}_c$ based on the above considerations, and the plastic *C.O.D.* is not calculable from the lateral contraction since $\delta = \epsilon p$.

The foregoing assumes that the plastic strain p is constant over a gage length ϵ. In reality this must be regarded as a simplified model and that p might better be defined as the average strain.

If a reliable functional relationship could be established between p and ϵ or between p and δ_p or between p and K_c by any method, empirical or otherwise, then the lateral contraction p could obviously be used to compute both δ_p and K_c. If a relation between δ_c and p can be directly established by experiment as in Fig. 1 (for HY 130), it follows that a functional relation between K_c and δ_p and between p and ϵ is thereby established also. Figure 1 represents experimental data taken by Mr. George Young and Mr. H. L. Smith of NRL. As shown in Fig. 1 the same functional relation does not hold for other materials even in identically dimensioned specimens, and it is rather to be expected that the relation would vary also with specimen design, work hardening coefficients, and whatever else determines the plastic flow pattern.

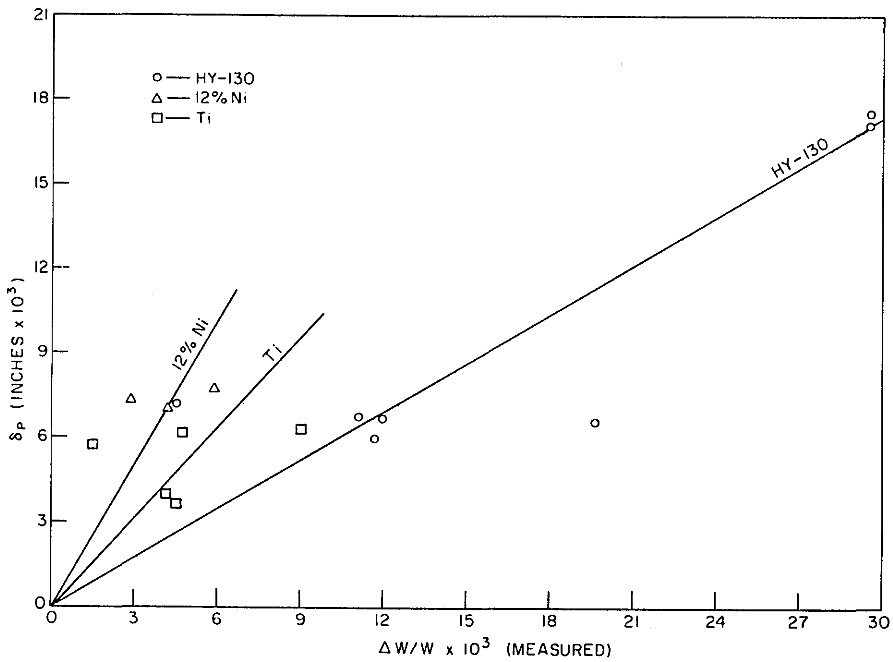

Fig. 1 - Crack opening displacement δ_p (calculated from measured $\mathscr{d}_c$) plotted against the lateral contraction $\Delta W/W$. The specimens were bend bars of identical size and shape loaded at three points

CONCLUSIONS

1. Physical limitations on the applicability of a previously published relation between the fracture mechanics intensity factor K and the Neuber stress factor were specified. Block K was substituted for script $\mathcal{K}/\sqrt{\pi}$.

2. The Neuber plastic particle size was shown to be four times the Irwin plasticity correction independent of whether plane stress or plane strain apply.

3. A formula for the plastic portion of the crack opening displacement $C.O.D.$ was derived and compared with the purely elastic portion of the $C.O.D.$ for an ideal model based on linear fracture mechanics. The Neuber plastic particle size was used in the derivation. The result was nearly the same but not identical with the formula first proposed by A. A. Wells using a different approach.

4. It was shown that the plastic lateral contraction at the tip of a crack cannot be derived from existing formulas for the $C.O.D.$; i.e., the use of lateral contraction to predict $C.O.D.$ or $\mathscr{d}_c$ is highly empirical at present.

5. Data for notched bend bars of identical size show that there are entirely different functional relations between $C.O.D.$ and lateral contractions depending on the material used.

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13. ABSTRACT

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